

Motion around the out-of-plane equilibrium points in the photogravitational Copenhagen elliptic restricted three-body problem with oblateness

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Abstract: This paper studies the motion of an infinitesimal mass near the out-of-plane equilibrium points (OEPs) in the elliptic restricted three-body problem (ER3BP) in the case of two equally heavy bodies (Copenhagen problem) where one of the two primaries is a radiation source and the other an oblate spheroid. We found, as in the photogravitational circular restricted three body problem, that the equations of motion of the three dimensional photogravitational ER3BP allow the existence of OEPs if the radiation parameter has negative values. There are two out of plane equilibria that lie in the (ξ, ζ) plane in symmetrical positions with respect to the (ξ, η) plane. The positions of the OEPs are affected by the parameters involved in the systems' dynamics. In particular, the positions change with increase in the radiation pressure, oblateness, eccentricity and semi-major axis of the orbits. As an application, the positions and linear stability of the problem are investigated numerically for the binary system B1534+12. The OEPs are found unstable.

Keywords: celestial mechanics; elliptic restricted three-body problem; ER3BP; radiation pressure; oblateness; Copenhagen problem; out-of-plane equilibrium points; OEPs; binary system; stability.

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8 Conclusions

The main conclusions of this work can be summarised as follows.

- 1 In the present study, the existence of OIEPs strongly depends upon the values of the radiation factor of the secondary body.
- 2 The stability analysis suggests that, in the present case of equal masses, the OIEPs of the system are linearly unstable.
- 3 In the absence of radiation of the secondary ($q_2 = 1$) and oblateness coefficient of the primary ($\mu_1 = 0$), the gravitational case is recovered: on the other hand, if the eccentricity is zero ($e = 0$) and the semi-major axis is unity ($a = 1$), the system is reduced to the CR3BP.

In our future work, we intend to study some features of this model, like the zero velocity curves and surfaces with some other type of perturbations.

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