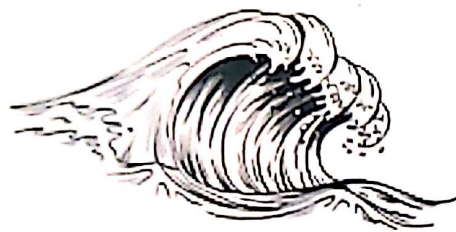


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Analysis, Geometry, Nonlinear Optimization and Applications



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Preface

Analysis, Geometry, Nonlinear Optimization and Applications publishes papers which are devoted to both classical as well as very current domains of research. Specifically topics treated within this book include optimization of control points, graph pursuit games, game theory models, Nash equilibrium, motion around equilibrium points in the photogravitational R3BP, quantitative methods, linearization methods, dynamic geometry, superquadratic functions, semicontinuous relations in relator spaces, generalized Cartan matrices, multivalued mappings, integral inequalities, norm inequalities, variational inequalities, variational inclusions, generalized Ostrowski and trapezoid type rules, stability of functional equations, hyperstability of ternary Jordan homomorphisms, nonlinear evolution PDEs in Biomechanics, Volterra integro-differential equations, interpolation functions and hypergeometric series, Caristi-Kirk theorems, metrical coercivity, low rank affine groups, wavelet transforms, wavelet fluctuation analysis and related subjects.

Effort has been made for the problems studied within this volume to have an interdisciplinary flavor as well as extend to a broad spectrum of domains, presenting the state of the art on the problems treated.

We would like to express our warmest appreciation to all the authors who contributed their valuable works to be published in this volume. We would also like to extend our sincere thanks to the staff of World Scientific Publ. Co. for their help throughout the preparation of this book.

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Chapter 29

Motion Around the Equilibrium Points in the Photogravitational R3BP under the Effects of Poynting–Robertson Drag, Circumbinary Belt and Triaxial Primaries with an Oblate Infinitesimal Body: Application on Achird Binary System

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In the present work, we study the motion of an oblate infinitesimal mass body near the equilibrium points (EPs) of the circular restricted three-body problem (R3BP) in which the radiation pressure, Poynting–Robertson (P–R) drag effect, and triaxiality of the two primary bodies are considered in the case where both of them are enclosed by a belt of homogeneous circular cluster of material points centered at the mass center of the system. We have found numerically that five or seven EPs may lie on the plane of motion depending on the values of the parameters of the system and have examined their stability character, too. In particular, the numerical exploration is performed using the binary system Achird to compute the positions of the equilibria and the eigenvalues of the characteristic equation. It is observed that the existence, location and number of equilibria of the problem depend on the values of the parameters of the problem. We have found both numerically and analytically that under constant P–R drag effect, collinear equilibrium solutions cease to exist. The linear stability of each equilibrium point is also studied and it is found that in the case where seven equilibria exist, the new point L_{N2} is always linearly stable while the other six are always linearly unstable. In the case where five equilibria exist, all of them are always linearly unstable due to P–R drag effect.

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