

Out-of-plane equilibria in the restricted five-body problem with radiation pressure

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Abstract: The photogravitational restricted five-body problem is employed to describe the motion of an infinitesimal test particle in the special case where two of the primaries are radiation sources. The four primaries m_i , $i = 0, \dots, 3$ three of which have equal masses ($m_1 = m_2 = m_3 = m$) are located at the vertices of an equilateral triangle, while the fourth one with a different mass m_0 is located at the centre of this configuration (Ollöngren's configuration). The fifth body of negligible mass moves in the resultant force field of the primaries and does not affect the motion of the four bodies (primaries). We consider that the central primary body m_0 and one of the peripheral bodies m_1 are radiation sources. The equilibrium points lying out of the orbital plane of the primaries as well as the allowed regions of motion as determined by the zero velocity curves are studied numerically. Finally, the stability of these points is also examined.

Keywords: celestial mechanics; out-of-plane equilibria; restricted five-body problem; RSBP; Ollöngren's problem; radiation pressure; zero velocity curves; ZVCs; stability.

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1 Introduction

The equilibrium solution of N -body celestial systems and related equilibrium points analysis have always been an attractive and important field of research, especially these days due to the detection of more than 2,500 extra-solar planetary systems (<http://exoplanet.eu/catalog/>). There is no doubt that one of the most well investigated

6 Discussions and conclusions

We studied the existence, location and stability of the out of plane equilibrium points of the R5BP when the four primaries three of which have equal masses are located at the vertices of an equilateral triangle (Lagrangian configuration), while the fourth one with a different mass is located at the centre of this configuration. As it is known, such points do not appear if only the gravitational forces are considered. We examined the case when the central primary m_0 and one of the peripheral bodies (m_1) are radiation sources. There are two out of plane equilibrium points that lie in the (x, z) plane in symmetrical positions with respect to the (x, y) plane. Our numerical exploration revealed how the positions of the critical points are influenced for various values of the radiation and mass parameters ($q_{0,1}$) and β correspondingly. We found that as the value of the radiation pressure and mass parameter increases, the positions of the out of plane equilibrium points are significantly affected. The effects of the parameters involved on the positions of the out of plane points are shown in Tables 1–6 (as well as in Figures 2–7). Furthermore, radiation pressure as well as mass parameter is also seen to have significant effects on the topology of the zero velocity curves in the (x, z) plane (Figures 8 and 9). Finally, the stability of the out of plane equilibrium points is completely determined by the roots of the characteristic equation (11). The stability of the system has been studied using numerical approach in order to express the partial derivatives clearly. The numerical investigation of these roots shows no case in which all the six roots are purely imaginary quantities. Consequently, the motion is unbounded, and thus unstable. Our result agrees with previous assertions of Papadouris and Papadakis (2013), Singh and Vincent (2015, 2016), and Arribas et al. (2016). To our knowledge, this is the first time that such a numerical investigation, regarding the out of plane equilibrium points, takes place in the Lagrangian R5BP and this is exactly the novelty as well as the importance of the current work. We hope that our investigation and the corresponding outcomes to be useful in connection with planetary system formation, satellite motion, etc.

Finally, periodic motion around the out of plane equilibrium points when some or all the primary bodies are sources of radiation we will present in a future article.

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