

Combined effects of radiation and oblateness on the existence and stability of equilibrium points in the perturbed restricted four-body problem

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Abstract: We study numerically the perturbed problem of four bodies, where an infinitesimal body is moving under the gravitational attraction of three primary bodies which move on circular orbits around their common centre of gravity, such that their configuration is always an equilateral triangle. The problem is perturbed in the sense that the dominant primary body m_1 is a radiation source while the second primary m_2 is an oblate spheroid, with masses of the two small primaries m_2 and m_3 taken to be equal. We investigate the effects of radiation and oblateness parameters on the existence and location of equilibrium points and their linear stability. The zero-velocity surfaces are also given. It is observed that under the perturbative effect of oblateness, collinear equilibrium points do not exist whereas the positions of the non-collinear equilibrium points are affected by the parameters. The stability of each points ($L_i, i = 1, \dots, 8$) is also studied.

Keywords: celestial mechanics; restricted four-body problem; R4BP; radiation; oblateness; equilibrium points; zero-velocity surfaces; ZVSs; stability

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Biographical notes: Jagadish Singh is a Professor at the Department of Mathematics, Ahmadu Bello University, Zaria, Nigeria. He received his PhD in 1985 and DSc in 2003 in Mathematics at B R A Bihar University, Muzaffarpur, India. He worked as a Research Associate under the Department of Science and Technology Project, New Delhi. He has been involved in both research and teaching since 1982. He has supervised several MSc and PhD

5 Discussion and conclusions

We have studied the perturbed restricted four-body problem when the primary bodies m_1 , m_2 and m_3 are always at the vertices of an equilateral triangle (Lagrangian configuration). The fourth particle in this system has negligible mass m with respect to the primaries, and its motion is perturbed by radiation pressure q_1 and oblateness coefficient A_2 from the primaries m_1 and m_2 , respectively. We studied the existence, location and stability of the equilibrium points as the radiation and oblateness parameters vary. The effects of the involved parameters on the positions of the equilibrium points are presented in Appendices A and B. These are shown graphically in Figures 4, 5, 6 and 7. It is found that for fixed values of oblateness coefficient A_2 and decreasing values of radiation pressure q_1 as well as for fixed values of radiation pressure q_1 and increasing values of oblateness coefficient A_2 , positions of the equilibrium points are significantly affected. The allowed regions of motion as determined by the zero-velocity surface are also given. Oblateness factor is seen to have significant effects on the topology of the zero velocity surface (Figure 2). Finally the stability investigation has been achieved according to inequalities (18)–(20). The numerical results provide that for fixed value of μ the linear stability of the equilibria does not change as the parameters of oblateness and radiation vary (i.e., L_i , $i = 1, \dots, 6$ are unstable while L_7 and L_8 are stable) whereas this situation changes as the mass parameter μ varies. Specifically the stability of L_1 is affected since there are intervals of μ where the inequalities (18)–(20) are satisfied simultaneously. It is remarkable to note that, equations of motion are unlike those obtained by Papadouris and Papadakis (2013) due to the appearance of oblateness coefficient of the second small primary. We observe that the perturbation given in the oblateness coefficient which is considered permits the disappearance of collinear equilibrium points contrary to Kumari and Kushvah (2014) results which has two collinear points. The positions of these points are different from those of Kumari and Kushvah (2014).

References

- Abdul Raheem, A. and Singh, J. (2006) 'Combined effects of perturbations, radiation, and oblateness on the stability of equilibrium points in the restricted three-body problem', *Astron. J.*, Vol. 131, No. 3, pp. 1880–1885.
- Abdul Raheem, A. and Singh, J. (2008) 'Combined effects of perturbations, radiation and oblateness on the periodic orbits in the restricted three-body problem', *Astrophys. Space Sci.*, Vol. 317, No. 1, pp. 9–13.
- Abouelmagd, E.I. and El-Shaboury, S.M. (2012) 'Periodic orbits under combined effects of oblateness and radiation in the restricted problem of three bodies', *Astrophys. Space Sci.*, Vol. 341, No. 2, pp. 331–341.
- Abouelmagd, E.I. and Sharaf, M.A. (2013) 'The motion around the libration points in the restricted three-body problem with the effect of radiation and oblateness', *Astrophys. Space Sci.*, Vol. 344, No. 2, pp. 321–332.
- Alvarez-Ramirez, M. and Vidal, C. (2009) 'Dynamical aspects of an equilateral restricted four-body problem', *Mathematical Problems in Engineering*, Vol. 2009, Article ID: 181360, pp. 1–23.
- Balogiannis, A.N. and Papadakis, K.E. (2013) 'Periodic solutions in the Sun-Jupiter-Trojan asteroid-spacecraft system', *Planetary and Space Science*, Vol. 75, pp. 148–157, DOI:10.1016/j.pss.2012.11.006.

- Baltagiannis, A.N., Papadakis, K.E. (2011) 'Equilibrium points and their stability in the restricted four-body problem', *Int. J. Bifurc. Chaos.*, Vol. 21, No. 8, pp.2179–2193.
- Burgos-Garcia, J. and Delgado, J. (2013) 'Periodic orbits in the restricted four-body problem with two equal masses', *Astrophys. Space Sci.*, Vol. 345, No. 2, pp.247–263.
- Ceccaroni, M. and Biggs, J. (2012) 'Low-thrust propulsion in a coplanar circular restricted four-body problem', *Celest. Mech. Dyn. Astron.*, Vol. 112, No. 2, pp.191–219.
- Gascheau, M. (1843) 'Examen d'une classe d'equations differentielles et application a un cas particulier du probleme des trois corps', *Comp. Rend.*, Vol. 16, pp.393–394.
- Ishwar, B. and Kushvah, B.S. (2006) 'Linear stability of triangular equilibrium points in the generalized photogravitational restricted three body problem with Poyting-Robertson drag', *Journal of Dynamical Systems and Geometric Theories*, Vol. 4, No. 1, pp.79–86.
- Kumari, B. and Kushvah, B.S. (2014) 'Stability regions of equilibrium points in restricted four-body problem with oblateness effects', *Astrophys. Space Sci.*, Vol. 349, No. 2, pp.693–704.
- Machuy, A.L., Prado, F.B.A. and Stuchi, T.J. (2007) 'Numerical study of the time required for the gravitational capture in the bi-circular four-body problem', *Adv. Space Res.*, Vol. 40, No. 1, pp.118–124.
- McCuskey, S.W. (1963) *Introduction to Celestial Mechanics*, Addison-Wesley, New York.
- Moeckel, R. (1994) 'Linear stability of relative equilibria with a dominant mass', *J. Dyn. Differ. Eqn.*, Vol. 6, No. 1, pp.37–51.
- Papadouris, J.P. and Papadakis, K.E., (2013) 'Equilibrium points in the photogravitational restricted four-body problem', *Astrophys. Space. Sci.*, Vol. 344, No. 1, pp.21–38.
- Routh, E.J. (1875) 'On Laplace's three particles with a supplement on the stability of steady motion', *Proc. Lond. Math. Soc.*, Vol. 6, pp.86–97.
- Santoprete, M. (2006) 'Linear stability of the Lagrangian triangle solutions for quasihomogeneous potentials', *Celest. Mech. Dyn. Astron.*, Vol. 94, No. 1, pp.17–35.
- Singh, J. and Ishwar, B. (1999) 'Stability of triangular points in the generalized photogravitational restricted three-body problem', *Bull. Astron. Soc. India.*, Vol. 27, No. 3, pp.415–424.
- Singh, J. and Vincent, E.A. (2015) 'Out-of-plane equilibrium points in the photogravitational restricted four-body problem', *Astrophys. Space Sci.*, Vol. 359, No. 1 [online] <http://doi:10.1007/s10509-015-2487-0> (accessed 01 September 2015).
- Tsirogiannis, G.A., Douskos, C.N. and Perdios, E.A. (2006) 'Computation of the Liapunov orbits in the photogravitational RTBP with oblateness', *Astrophys. Space Sci.*, Vol. 305, No. 4, pp.389–398.
- Vishnu, N., Sudheer, R. and Sharma, R.K. (2008) 'Effect of oblateness and radiation pressure on angular frequencies at collinear points', *Astrophys. Space Sci.*, Vol. 318, No. 3, pp.161–168.