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Chapter

Periodic Solutions Around the Out-of-Plane Equilibrium Points in the Restricted Three-Body Problem with Radiation and Angular Velocity Variation

Chapter | First Online: 30 September 2020

pp 251–275 | [Cite this chapter](#)



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About this chapter

Springer Optimization and Its Applications 167

Themistocles M. Rassias
Panos M. Pardalos *Editors*

Nonlinear Analysis and Global Optimization

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Preface

Nonlinear Analysis and Global Optimization is devoted to the study of a broad area of research with strong interdisciplinary flavor in which optimization plays an important role, as well as to the study of topics that are applied toward the investigation of optimization problems. More specifically, the present book treats topics on set-valued analysis, mixed concave–convex sub-superlinear Schrödinger equation, Schrödinger equations in nonlinear optics, exponentially convex functions, optimal lot size under the occurrence of imperfect quality items, generalized equilibrium problems, artificial topologies on a relativistic spacetime, equilibrium points in the restricted three-body problem, optimization models for networks of organ transplants, network curvature measures, error analysis through energy minimization and stability problems, Ekeland variational principles in 2-local Branciari metric spaces, frictional dynamic problems, norm estimates for composite operators, operator factorization and solution of second-order nonlinear difference equations, degenerate Kirchhoff-type inclusion problems, and more.

We would like to express our deep thanks to all the authors of contributed book chapters for participating in this collective effort. We would also like to express our thanks to the staff of Springer for their help throughout the preparation of this book.

Athens, Greece

Gainesville, FL, USA

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Periodic Solutions Around the Out-of-Plane Equilibrium Points in the Restricted Three-Body Problem with Radiation and Angular Velocity Variation



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Abstract In the present work, we study the motion of an infinitesimal body near the out-of-plane equilibrium points of the restricted three-body problem in which the angular velocity of the two primary bodies is considered in the case where both of them are sources of radiation. Firstly, these equilibria are determined numerically, and then the influence of the system parameters on their positions is examined. Due to the symmetry of the problem, these points appear in pairs and, depending on the parameter values, their number may be zero, two, or four. The linear stability of the out-of-plane equilibrium points is also studied, and it is found that there are cases where they can be stable. In addition, periodic motion around them is investigated both analytically and numerically. Specifically, the Lindstedt–Poincaré method is used in order to obtain a second order analytical solution, while the families emanating from the out-of-plane equilibrium points are finally computed numerically either in case where the corresponding equilibrium points are stable or unstable. For the numerical computation of a three-dimensional periodic orbit, we apply known unconstrained optimization methods to an objective function that is formed by the respective periodicity conditions that have to be fulfilled.

MSC 70F07, 70F15, 70M20, 70K42

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T. M. Rassias, P. M. Pardalos (eds.), *Nonlinear Analysis and Global Optimization*,
Springer Optimization and Its Applications 167,
https://doi.org/10.1007/978-3-030-61732-5_11

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