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About this chapter



On the Stability of the Triangular Equilibrium Points in the Photogravitational R3BP with an Oblate Infinitesimal and Triaxial Primaries for the Binary Lalande 21258 System



Jessica Mrumun Gyegwe, Aguda Ekele Vincent, and Angela E. Perdiou

Abstract In the framework of the planar circular restricted three-body problem (R3BP), we explore the effects of oblateness of the infinitesimal mass body as well as radiation pressure and triaxiality of the two primaries on the position and stability of the triangular equilibrium points (TEPs). It is found that all the involved parameters affect the positions and stability of these points. Specifically, it has been shown that TEPs are stable for $0 < \mu < \mu_c$ and unstable for $\mu_c \leq \mu \leq 1/2$, where μ_c denotes the critical mass parameter which depends on system's parameters. In addition, all the parameters of the bigger primary, except that of triaxiality, have destabilizing tendencies resulting in a decrease in the size of the region of stability. Finally, we justify the relevance of the model in astronomy by applying it to the binary Lalande 21258 system for which the equilibrium points have been seen to be unstable.

MSC 70F07, 70F15, 70M20, 70K42

1 Introduction

The restricted three-body problem (R3BP) has to do with three bodies or masses. Two of the bodies have tangible masses and are known as the primaries while

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Fig. 4 (Colour figure online) Positions of $L_{4,5}$ of Lalande 21258 for combined effects of instability of both primaries and oblateness (case 8 of Table 2). blue colour $\phi_1 = 0.008$, $\psi_2 = 0.006$, $\psi_1 = 0.006$, $\psi_2 = 0.004$, and $\Lambda_3 = 0.01$; red colour $\phi_1 = 0.03$, $\psi_2 = 0.02$, $\psi_1 = 0.015$, $\psi_2 = 0.01$, and $\Lambda_3 = 0.05$; black colour $\phi_1 = 0.05$, $\psi_2 = 0.035$, $\psi_1 = 0.03$, $\psi_2 = 0.02$, and $\Lambda_3 = 0.1$

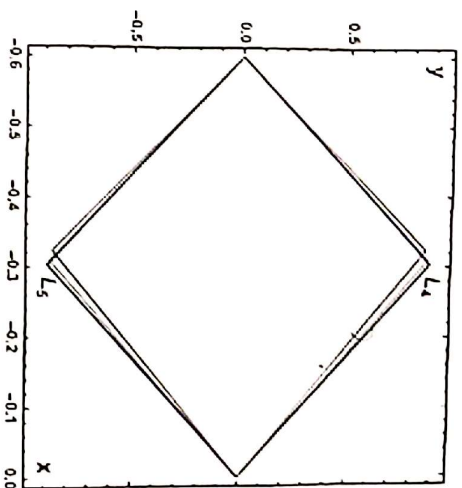


Table 3 The eigenvalues $\lambda_{1,2}$, $\lambda_{3,4}$ of Eq. (10) for Lalande 21258 system

ϕ_1	ϕ_2	ψ_1	ψ_2	Λ_3	$\lambda_{1,2}$	$\lambda_{3,4}$
0.008	0.006	0.006	0.004	0.01	$-0.513933 \pm 0.868413i$	$0.513933 \pm 0.868413i$
0.03	0.02	0.015	0.01	0.05	$-0.574577 \pm 0.879048i$	$0.574577 \pm 0.879048i$
0.05	0.035	0.03	0.02	0.1	$-0.642373 \pm 0.890378i$	$0.642373 \pm 0.890378i$
0.09	0.05	0.04	0.03	0.15	$-0.684338 \pm 0.899161i$	$0.684338 \pm 0.899161i$

radiation, i.e. $q_1 = q_2 = 1$, $\Lambda_3 = 0$, then we will get the results obtained by Elshaboury et al. [17]. Also, in the presence of the Coriolis and centrifugal forces when the two primaries are oblate spheroids, namely $q_1 = q_2 = 1$, $\phi_1 = \phi_2$, $\psi_1 = \psi_2$, our results will tally with those given by Abouelmagd and Guirao [1]. Finally, in the absence of radiation forces when the smaller primary body is an oblate spheroid and if the Coriolis and centrifugal forces are taken also into account, i.e. $q_1 = q_2 = 1$, $\Lambda_3 = 0$, $\psi_1 = \psi_2$, our results are in agreement with those of Singh and Begha [29]. In a future correspondence, we intend to extend our results by studying the families of short and long periodic orbits emanating from triangular equilibrium points. Furthermore, a natural extension of our present work would be also to incorporate the Poynting–Roberson relativistic correction in the radiation force and study how this strengthened force affects the location and stability of the respective equilibria.

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