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Existence and Stability of Equilibrium Points Under the Influence of Poynting–Robertson and Stokes Drags in the Restricted Three-Body Problem



Aguda Ekele Vincent and Angela E. Perdiou

Abstract In the framework of the circular restricted three-body problem, the dynamical effects of Stokes and Poynting–Robertson (P–R) drag forces on the existence, location, and stability of equilibrium points are investigated. It is found that under constant effects of P–R and/or Stokes drags, collinear equilibrium points cease to exist, but there are in the absence of the perturbing forces. The problem admits five non-collinear equilibrium points, and it is seen that the perturbing forces have significant effects on their positions. The linear stability of the equilibrium points is also studied in certain cases, and it is found that the stability of some of these points significantly depends on the perturbing forces. More precisely, the motion of the infinitesimal body near the non-collinear equilibrium points is unstable under the effect of both kinds of perturbing forces except from the equilibria L_4 and L_5 for which is stable only for Stokes drag effect, namely, the remaining parameter that corresponds to P–R drag is fixed to zero. We may conclude, therefore, that the P–R effect destroys stability of the equilibrium points.

MSC 70F07; 70F15; 70K20, 70K42

1 Introduction

The circular restricted three-body problem (CR3BP) consists of two finite bodies, known as primaries, which rotate in circular orbits around their common center of mass and a massless body that moves in the plane of motion of the primaries under their gravitational attraction and does not affect their motion. The CR3BP

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dominant, third when the Stokes drag is dominant, and fourth when both Stokes drag and P-R drag are in action together. In the case where the P-R drag and/or Stokes drag were considered, our numerical investigation for the number and location of the equilibria showed that five non-collinear equilibrium points exist in contrast to the absence of the aforementioned perturbing forces, i.e., the classical case, where five equilibrium points also exist, but three of them lie on the axis joining the primaries, while the rest two form in the plane of motion equilateral triangles with the primaries.

Additionally, it was observed that the involved parameters of the problem not only affect the positions of the corresponding equilibria but also influence their stability as well. For the determination of the stability of the infinitesimal body's motion around the obtained equilibria, we linearized the governing equations of motion around them. For all the considered cases, it was found that all equilibria are unstable due to the existence of a positive real root or a complex root with positive real part except for the equilibria L_4 and L_5 in the case of Stokes drag where we got complex roots with negative real parts, which means that these points are stable. Therefore, we conclude that the stability of the non-collinear equilibrium points L_4 and L_5 is not affected from the Stokes drag force of the primaries, and the motion in their vicinity remains stable for the value of mass parameter considered here. Also, contrary to the classical restricted three-body problem where the three collinear points are generally unstable and the triangular points are linearly stable for sufficiently small ratio of the two masses, we observed that for the problem under consideration all the equilibrium points are unstable due to the presence of the P-R drag effect. The instability of the equilibrium points agrees with the results existing in the literature when the Stokes drag force is not considered.

References

1. V. Szebehely, *Theory of orbits: The restricted problem of three bodies*, Academic press, New York (1967)
2. C. Marchal, *The three body problem*, Studies in Astronautics -4, Elsevier, Amsterdam (1990)
3. R. Farquhar, 'The flight of ISEE-3/ICE: Origins, mission history, and a legacy', *J. Astronaut. Sci.*, **49**, 23–73 (2001)
4. P. Sharer and T. Harrington, 'Trajectory Optimization for the Ace Halo Orbit Mission', In *AIAA/AAS Astrodynamics Specialist Conference*, San Diego, California, July 29-31, Paper AAS 96-3601 (1996)
5. M. Giard and L. Montier, 'Investigating clusters of galaxies with Planck and Herschel', *Astrophys. Space Sci.*, **290**, 159–166 (2004)
6. P. Overti and A. Vienne, 'An upgraded theory for Helene, Telesto, and Calypso', *Astron. Astrophys.*, **397**, 353–359 (2003)
7. P. Verrier, T. Waters and J. Sieber, 'Evolution of the L_1 halo family in the radial solar sail circular restricted three-body problem', *Celest. Mech. Dyn. Astr.*, **120**, 373–400 (2014)
8. S.M. Elshaboury, E.I. Abouelmagd, V.S. Kalantonis and E.A. Perdios, 'The planar restricted three-body problem when both primaries are triaxial rigid bodies: Equilibrium points and periodic orbits', *Astrophys. Space Sci.*, **361**, 315 (2016)

9. N. Pathak, E.I. Abouelmagd and V.O. Thomas, On higher order resonant periodic orbits in the photo-gravitational planar restricted three-body problem with oblateness. *J. Astronaut. Sci.* **66**, 475–505 (2019)
10. E.E. Zotos and K.E. Papadakis, Orbit classification and networks of periodic orbits in the planar circular restricted five-body problem. *Int. J. Nonlin. Mech.* **111**, 119–141 (2019)
11. F. Gao and R. Wang, Bifurcation analysis and periodic solutions of the HD 191408 system with triaxial and radiative perturbations. *Universe* **6**, 35 (2020)
12. V.S. Kalantonis, Numerical Investigation for Periodic Orbits in the Hill Three-Body Problem. *Universe*, **6**, 72, (2020)
13. C.D. Murray, Dynamical effects of drag in the circular restricted three body problems. I. Location and stability of the Lagrangian equilibrium points. *Icarus* **112**, 465–484 (1994)
14. C. Beaugé and S. Ferraz-Mello, Resonance trapping in the primordial solar nebula: the case of a Stokes drag dissipation. *Icarus* **103**, 301–318 (1993)
15. B. Sicardy, C. Beaugé, S. Ferraz-Mello, D. Lazzaro and F. Roques, Capture of grains into resonances through Poynting–Robertson drag. *Celest. Mech. Dyn. Astron.* **57**, 373–390 (1993)
16. A. Celletti, L. Stefinelli, E. Lega and C. Froeschlé, Some results on the global dynamics of the regularized restricted three-body problem with dissipation. *Celest. Mech. Dyn. Astron.* **109**, 265–284 (2011)
17. J. Singh and A. Aminu, Instability of triangular libration points in the perturbed photogravitational R3BP with Poynting–Robertson (P-R) drag. *Astrophys. Space Sci.* **351**, 473–482 (2014)
18. M. Jain and R. Aggarwal, Existence and stability of non-collinear libration points in restricted three body problem with Poynting Robertson light drag effect. *Int. J. Math. Trends and Technol.* **19**, 20–33 (2015b)
19. E.E. Zotos and E.L. Dubeibe, Orbital dynamics in the post Newtonian planar circular restricted Sun-Jupiter system. *Int. J. Mod. Phys. D* **27**, 1850036 (2018)
20. E.A. Vincent and E.A. Perdiou, Poynting–Robertson and oblateness effects on the equilibrium points of the perturbed R3BP: Application on Cen X-4 binary system. In Rassias, Th.M. (Ed.) *Nonlinear Analysis, Differential Equations, and Applications*, Springer Optim. Its Appl. **173**, Springer, Cham (2021), in press
21. V.V. Rudzievskii, The photogravitational restricted problems of three bodies. *Astron. J.* **27**, 250–256 (1950)
22. J.E.L. Simmons, A.J.C. McDonald and J.C. Brown, The restricted 3-body problem with radiation pressure. *Celes. Mech.*, **35**, 145–187 (1985)
23. A.L. Kunitsyn and E.N. Polyakhova, The restricted photogravitational three-body problem. A modern state, *Astron. Astrophys. Trans.*, **6**, 283–293 (1995)
24. V.S. Kalantonis, E.A. Perdios and O. Ragos, Asymptotic and periodic orbits around L_4 in the photogravitational restricted three-body problem. *Astrophys. Space Sci.* **301**, 157–165 (2006)
25. E.A. Perdios, V.S. Kalantonis and C.N. Douskos, Straight-line oscillations generating three-dimensional motions in the photogravitational restricted three-body problem. *Astrophys. Space Sci.* **314**, 199–208 (2008)
26. D.G. Yirmoz, J.P.S. Cuatrecasas and C.R. McInnes, Passive sorting of asteroid material using solar radiation pressure. *J. Guid. Control Dyn.* **37**, 1223–1235 (2014)
27. H. Baoyin and C.R. McInnes, Solar sail halo orbits at the Sun–Earth artificial L_1 point. *Celest. Mech. Dyn. Astron.* **94**, 155–171 (2006)
28. S. Gong, J. Li and H. Baoyin, Analysis of displaced solar sail orbits with passive control. *J. Guid. Control. Dyn.* **31**, 782–785 (2008)
29. Yu.A. Chernikov, The photogravitational restricted three-body problem. *Soviet Astronomy-AJ* **14**, 176–181 (1970)
30. D.W. Schuerman, Influence of the Poynting–Robertson effect on triangular points of the photogravitational restricted three-body problem. *Astrophys. J.* **238**, 337–342 (1980)
31. O. Ragos and F.A. Zafiropoulos, A numerical study of the influence of the Poynting–Robertson effect on the equilibrium points of the photogravitational restricted three-body problem. I. Coplanar case. *Astron. Astrophys.* **300**, 568–578 (1995)

32. J. Burns, P. Lamy and S. Soter, Radiation forces on small particles in the Solar system. *Icarus* **40**, 1–48 (1979)
33. J. Singh and A.M. Simeon, Motion around the triangular equilibrium points in the circular restricted three-body problem under triaxial luminous primaries with Poynting-Robertson drag. *Int. Front. Sci. Lett.* **12**, 1–21 (2017)
34. J. Singh, J. and T.O. Anuda, Stability analysis of triangular equilibrium points in restricted three-body problem under effects of circumbinary disc, radiation and drag forces. *J. Astrophys. Astr.* **40**, 5 (2019)
35. A. Umar and A.A. Hussain, Impacts of Poynting-Robertson drag and dynamical flattening parameters on motion around the triangular equilibrium points of the photogravitational ER3BP. *Adv. Astron.* vol. **2021**, Article ID 6657500 (2021)
36. M. Jain and R. Aggarwal, A study of non-collinear libration points in restricted three body problem with Stokes drag effect when smaller primary is an oblate spheroid. *Astrophys. Space Sci.* **358**, 51–58 (2015a)
37. J.C. Liou, H.A. Zook, and A.A. Jackson, Radiation pressure, Poynting-Robertson drag, and solar wind drag in the restricted three-body problem. *Icarus* **116**, 186–201 (1995)
38. C.D. Murray and S.F. Dermott, *Solar System Dynamics*. Cambridge University Press, Cambridge (1999)
39. A.N. Baltagiannis and K.E. Papadakis, Equilibrium points and their stability in the restricted four-body problem. *Int. J. Bifurc. Chaos* **21**, 2179–2193 (2011)
40. M. Arribas, A. Abad, A. Elise and M. Palacios, Out-of-plane equilibria in the symmetric collinear restricted four-body problem with radiation pressure. *Astrophys. Space Sci.* **361**, 210–280 (2016)
41. A.E. Vincent, J.J. Taura and S.O. Omale, Existence and stability of equilibrium points in the photogravitational restricted four-body problem with Stokes drag effect. *Astrophys. Space Sci.* **364**, 183 (2019)