

3-D Simulation for Assessing Projectile's Motion with or without Air Resistance

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ABSTRACT

A Model was developed using JAVA Programming Language; this package simulates the motions of projectiles in air with air resistivity put into considerations or air resistance being ignored. Since air blows over the surface of the earth, thus Air resistance is said to be the linear function of the velocity. The model is based on the equations governing the trajectory of projectiles in motion; it is also coupled with arrows showing the direction of the wind on a 3-dimensional plane. With the use of a computer system, the motion of objects in real life situations involving projectiles with or without air resistance can be simulated with this model. It is a good tool for the teaching of kinematical related course in tertiary institutions.

Keywords:-Simulation, Projectile, Model, Air resistance, Trajectory, Velocity, Acceleration, Kinematics, Aerial Warfare

INTRODUCTION

Due to the incessant increase in Terrorisms, show of powers, dominations, religious extremism, wars, contentions, and other outrageous excesses; there had been the need for the development of an accurate and tested space technologies used in aerial warfare (e.g. projectiles, like cannon for launching missiles, jet bombers, warships, guided missiles). For a body or system of bodies passing through space, can be subjected to a kinematical analysis for accurate determination of its position.

Kinematics, branch of physics and subdivision of classical mechanics concerned with the geometrically possible motion of a body or system of bodies without the considerations of forces involved (i.e. causes and effects of motions). The aims of kinematics are to

provide a description of the spatial position of bodies or system of material particles, the rate at which particles are moving (velocity), and the rate at which their velocity is changing, (acceleration). For a particle moving on a straight path, a list of positions and corresponding times would constitute a suitable scheme for describing the motion of the particle. A continuous description would require a mathematical formula expressing position in terms of time [1]. When a particle moves on a curved path, a description of its position becomes more complicated and requires two or three dimension.

Usually, projectile is considered to be a particle. A particle is defined as an object that is small compared to the space in which it travels, and it occupies a single point in space. It is usually subjected to kinematic analysis. Problems involving

projectile motion are always analyzed either in two dimensions or in three dimension i.e. horizontal, vertical, and inclined plane; (Length, Breadth and height) [2]. Motion parameter like the time of flight, the maximum height attains, the total time of travel and range can be obtained from either or both components. Projectile motion can be studied with or without the effect of air resistance.

When the effect of air resistance is ignored, projectile motion becomes relatively easy and moves in a parabolic path. Results obtained in this case are not valid for real life situation [3-4].

When air resistances are considered, more complex mathematical equations which are even difficult to analyze and solve are obtained. This is due to the fact that air properties like speed, density, temperature and pressure amongst others, have no clearly defined variation with time and space.

Ideally, to have an accurate description and obtain solution of practical or real life problems, the effect of air resistance must be considered. Doing this would require one to solve problems by numerical integration employing the use computer machines. Researchers have postulated that a few special problems can be treated by a simple mathematical procedure, the requirement for such been that the velocity is between 30.5m/s and 305m/s [5].

It has been reported that, a spherical object that is stable while in motion will give a trajectory in the same cross-section in the direction of motion. And if this objects moves through a calm air, having no strong updrafts, the air resistance that is formed during the flight is at all times tangent to the trajectory and it is always the motion of the objects. [6]

In the recent past, remarkable efforts have been made in the attempt to model the

motion of projectiles.

The projectile of a cricket ball in motion was modelled [7]. In the study, the cricket ball was subjected to a linear drag force as produced by the air resistance, and employing the perturbation expansion technique, the author was able to solve the output trajectory motion. A cricket ball stroked into the outfield, the improvement in accuracy resulting from including more conditions was studied in the approximate range equation. While imagining a cricket match situation where the batsman hits a straight drive back and passing the bowler for a six, peradventure; the ball leaves the bar at 40m/s at an angle of 30 degrees, it will clear a boundary line 90meters away.

A computer program was also developed to solve the equation of projectiles in the absence of air resistance [8]. This program gives more insight into complex systems and particle solution using computer animation technique (CAT) as displayed by the graphical user interface (GUI). This technique is used to create real effect, simulate this effect, check for variations, relationships existing between parameters as well as between quantities; those that would be impossible to view in the laboratory or by non-animation technique (NAT). (Simulation of various machines before the actual productions). This technique can also be extended to producing images from scientific data [7].

In the present work, a program was developed to simulate the motion of projectiles; considering a 3-dimensional analyses with or without the effect of air resistance, it will be able to show the direction of the air resistance per each launch angle, draw the trajectory of objects, solve the motion parameters and as well display the results on the monitor, it will likewise be able to transform from 3-D to 2-D. This program was developed using the JAVA Programming Language

with its WORA abilities (i.e. Write-Once and Run Anywhere), the program can run on any operating system and it requires a little of the system resources.

This work was motivated by a Nigerian military mishap which claimed the lives of over 100 people including medical personnel in an internally displaced people (IDP) camp when a missile was accidentally dropped in the camp. (Premium times, 2017). Hence a need for proper analysis of the system (putting all possible factors into considerations) before launching.

THEORETICAL BACKGROUNDS FOR PROJECTILES

When analyzing projectile motions, the ground is taken to be a plane surface. The range covered is relatively short and the trajectory is relatively shallow, such that acceleration due to gravity g may be taken as constant. g is normal to the plane representing the earth surface and the rotation of the earth is neglected. Lift force is neglected, although this has been shown to be significant, [8]. The mass of the object is neglected. There is an all-round or multi-dimensional force of gravity on the particle during the motion. The motion of the wind is tangent to trajectory, since air moves over the surface of the earth.

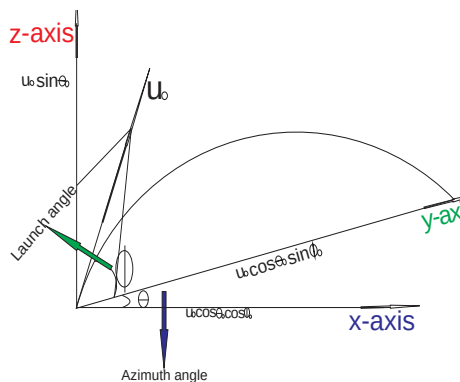


Fig.1:-Diagrammatical representation of projectile on a 3-D plane

Resolving the initial velocity U , into its components

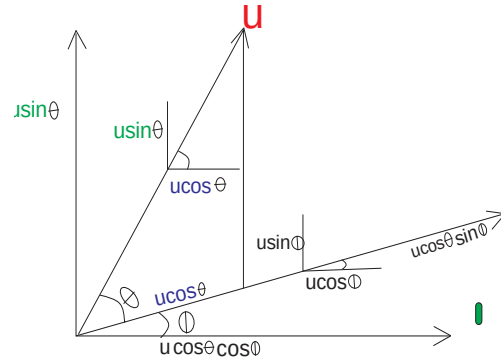


Fig.2:-Analysis of a projectile on a 3-D plane

$$u_{xo} = u_o \cos \Theta_o \cos \Phi_o \quad (1)$$

$$u_{yo} = u_o \cos \Theta_o \sin \Phi_o \quad (2)$$

$$u_{zo} = u_o \sin \Theta_o \quad (3)$$

Similarly, if the air resistance has similar components, the magnitude in speed R ; have the following values:

$$u_{xr} = R \cos \Theta_r \cos \Phi_r \quad (4)$$

$$u_{yr} = R \cos \Theta_r \sin \Phi_r \quad (5)$$

$$u_{zr} = R \sin \Theta_r \quad (6)$$

Velocity components with Air Resistance will be

$$u_x = u_o \cos \Theta_o \cos \Phi_o + R \cos \Theta_r \cos \Phi_r \quad (7)$$

$$u_y = u_o \cos \Theta_o \sin \Phi_o + R \cos \Theta_r \sin \Phi_r \quad (8)$$

$$u_z = u_o \sin \Theta_o + R \sin \Theta_r \quad (9)$$

Motion Parameters on a 3-Dimensional Plane, (without Air Resistance)

Time of flight, and Total time, $T = 2t$.

$$x = UT \quad (10)$$

$$u = u_o \cos \Theta_o \cos \Phi_o \quad (11)$$

$$t = \frac{u_o \cos \Theta_o \cos \Phi_o}{g} \quad (12)$$

$$x = u_o \cos \Theta_o \cos \Phi_o * \frac{2(u_o \cos \Theta_o \cos \Phi_o)}{g} \quad (13)$$

$$\text{Using the expression } \cos^2 \Theta = 1 - \sin^2 \Theta \quad (14)$$

$$x = \frac{2(u_o^2 \cos^2 \Phi_o - u_o^2 \sin^2 \Theta_o \cos \Phi_o)}{g} \quad (15)$$

Maximum height, H

$$H = \frac{u_o^2 \sin^2 \Phi_o \cos^2 \Theta_o}{2g} \quad (16)$$

Horizontal Distance covered, (Range)

$$s = \frac{u_o^2 \sin 2\Theta_o \cos \Phi_o}{g} \quad (17)$$

Motion Parameters on 3-Dimensional (With Air Resistance).

Time of Flight, t

Using the first equation of motion

$$T = \frac{2(u_o \cos \Theta_o \cos \Phi_o + R \cos \Theta_r \cos \Phi_r)}{g} \quad (18)$$

$$x = UT$$

$$t = \frac{u_o \cos \Theta_o \cos \Phi_o + R \cos \Theta_r \cos \Phi_r}{g} \quad (19)$$

$$u = u_o \cos \Theta_o \cos \Phi_o + R \cos \Theta_r \cos \Phi_r \quad (20)$$

$$\frac{2(u_o^2 \cos^2 \Theta_o \cos^2 \Phi_o + Ru_o \cos \Theta_o \cos \Theta_r \cos \Phi_r \cos \Phi_o)}{g} \quad (21)$$

Maximum Height Attained.

$$\text{Using } H = \frac{v^2 - u^2}{2g} \quad (22)$$

$$H = \frac{(u_o \sin \Theta_o + R \sin \Theta_r)^2}{2g} \quad (23)$$

Horizontal Distance Travelled (Range)

T = total time of flight, taken to travel from the plane of projection and to return back to the plain of projection again.

Mathematically, given by $T = 2t$

$$S = \frac{(v + u)}{2} * t \quad (24)$$

$$s = \frac{u_o^2 \sin 2\Theta_o \cos \Phi_o + Ru_o \sin \Theta_o \cos \Theta_r \sin \Phi_r}{g}$$

Hence the equation of the Trajectory on Z-X axes is given

$$x \tan \Theta_o \cos \Phi_o - \frac{gx^2}{(2u_o^2 \cos^2 \Theta_o \cos^2 \Phi_o)} \quad (25)$$

Hence the equation of the Trajectory on Y-X axes is given

$$x \tan \Phi_o - \frac{gx^2}{(2u_o^2 \cos^2 \Theta_o \cos^2 \Phi_o)} \quad (26)$$

Speed U, without air resistance is given as U_o and with air resistance is given as u_r

$$u_o = u_o \cos \Theta_o \cos \Phi_o \hat{i} + u_o \cos \Theta_o \sin \Phi_o \hat{j} + u_o \sin \Theta_o \hat{k} \quad (27)$$

$$u_r = u_r \cos \Theta_r \cos \Phi_r \hat{i} + u_r \cos \Theta_r \sin \Phi_r \hat{j} + u_r \sin \Theta_r \hat{k} \quad (28)$$

Thus, the position of the particle with air resistance is given thus

$$(u_o \cos \Theta_o \cos \Phi_o + u_r \cos \Theta_r \cos \Phi_r) \hat{i} + (u_o \cos \Theta_o \sin \Phi_o + u_r \cos \Theta_r \sin \Phi_r) \hat{j} + (u_o \sin \Theta_o + u_r \sin \Theta_r) \hat{k} \quad (29)$$

PHYSICS OF THE SOFTWARE

The 3-D Simulative Model was developed using JAVA programming Language. The model converts the projectile's equations into codes by the JAVA C compiler and then saved as .CLASS file, this .CLASS file contain source codes in the form of JAVA byte code, the byte code is then acted upon by the JAVA VIRTUAL MARCHINE (JVM) which then converts the byte code into *.obj files and this *.obj is used to generate the output. The output is software primarily design to illustrate the behavior of spherical objects (projectiles) in air per angle per direction. Respect to this, the animation window can display up to eight projectiles. Motion quantities like the initial velocity, launch angles and the magnitude of the air resistance are all supplied by the user for each of the projectiles. Once these values are inputted into the JTextFields provided, the simulation for each launched angle can then be viewed on the monitor. In addition, other motion parameters like the range, maximum height, time of flight, time taken to reach the maximum height is also computed and the trajectories are also generated and the result can also be printed. The trajectories displayed are all

based on the equation of motion mention in the preceding sections of this work.

It incorporates all the features of modern day programming, these include a very comprehensive graphical representation, user-friendly interface, nerve calming colors, time guarded plot or simulation

(i.e. can be paused, play, reversed, and forwarded), direction indicating arrows, two-dimensional sub-axes (transformation from 3-axes to 2-axes), clear plot button (removes the sub-axes), and clear view button (this removes all the plot on the main window). The user interface designed for this program is shown below.

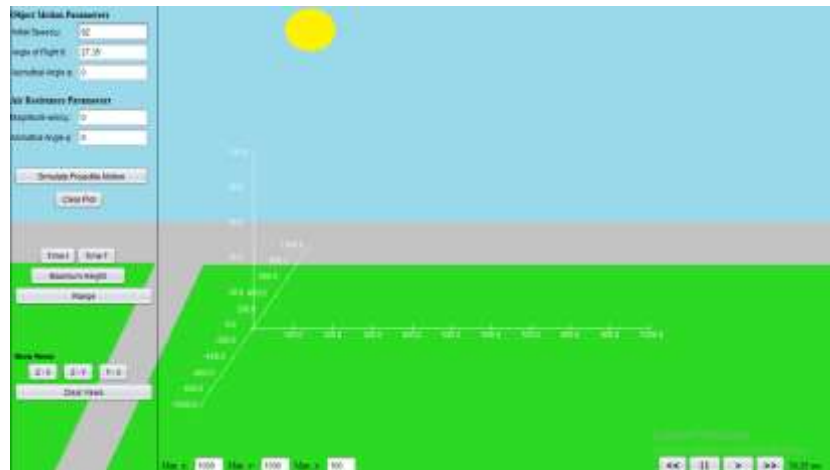


Fig.3:-The graphical user interface (animation window), for projectile simulation software

The animation window screen has a menu bar from which different option can be selected. In the JTextFields labeled initial speed μ , angle of flight θ , azimuth Φ , magnitude of the wind μ_0 , the indicated parameters of the projectiles are supplied by the user, after which the button with the caption “simulate projection motion” button is clicked; thereby the trajectory is

viewed. The x , y , and z axes can then be adjusted to suit the size limits of the users “Visual Display Unit (Monitor) screen.

And once all the motion parameters are inputted, the user is to click on the “simulate projectile motion”, then the simulation can then be seen.

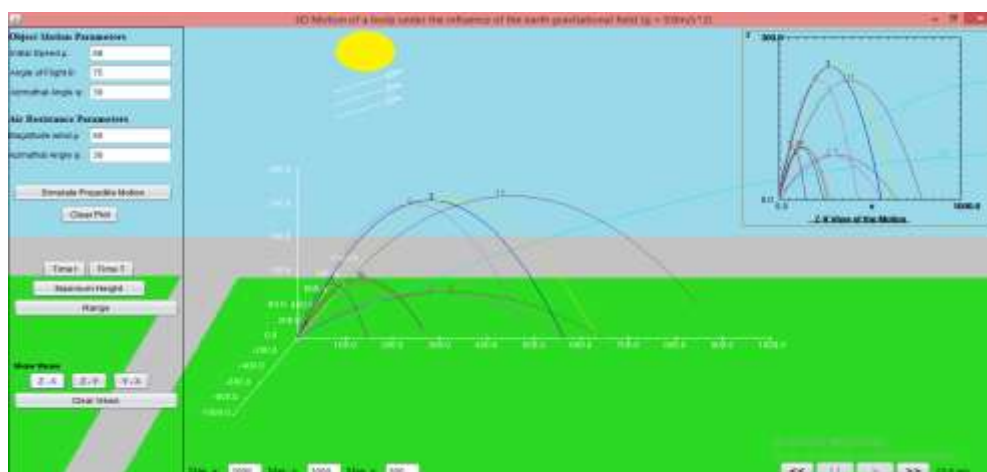


Fig.4:-“Multiple projectiles at various angles, air resistivity and direction indicating arrows” with a 2-D sub-axis

CONCLUSION

The program made a very low demand on computer resources, it gives the advantages of a modern but calm graphical user environment and it is truly a computer assisted tool to assist student in the understanding the trajectory of objects as well as helping both applied scientists and engineers understands real life situations.

It helps to understand and visualize the possible outcome in the application of projectiles, to real life situation by helping us to understand the relationship between the projectile's parameters and its quantities and how they affect one another, the program reveals a proportional relationship between the velocity and the Range, proportional relationship between the projecting angle and the pitch, at constant velocity and when projecting angle $\leq 60^\circ$ and the horizontal distance covered increases and when it $\geq 60^\circ$, the horizontal distance covered decreases.

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